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Diploma Programme

Mathematical studies SL guide

First examinations 2014

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IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:

Inquirers They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.

Knowledgeable They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.

Thinkers They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.

Communicators They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.

Principled They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.

Open-minded They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.

Caring They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.

Risk-takers They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.

Balanced They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.

Reflective They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

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Purpose of this document

This publication is intended to guide the planning, teaching and assessment of the subject in schools. Subject teachers are the primary audience, although it is expected that teachers will use the guide to inform students and parents about the subject.

This guide can be found on the subject page of the online curriculum centre (OCC) at http://occ.ibo.org, a password-protected IB website designed to support IB teachers. It can also be purchased from the IB store at http://store.ibo.org.

Additional resources

Additional publications such as teacher support materials, subject reports, internal assessment guidance and grade descriptors can also be found on the OCC. Specimen and past examination papers as well as markschemes can be purchased from the IB store.

Teachers are encouraged to check the OCC for additional resources created or used by other teachers. Teachers can provide details of useful resources, for example: websites, books, videos, journals or teaching ideas.

Acknowledgment

The IB wishes to thank the educators and associated schools for generously contributing time and resources to the production of this guide.

First examinations 2014

The Diploma Programme

The Diploma Programme is a rigorous pre-university course of study designed for students in the 16 to 19 age range. It is a broad-based two-year course that aims to encourage students to be knowledgeable and inquiring, but also caring and compassionate. There is a strong emphasis on encouraging students to develop intercultural understanding, open-mindedness, and the attitudes necessary for them to respect and evaluate a range of points of view.

The Diploma Programme hexagon

The course is presented as six academic areas enclosing a central core (see figure 1). It encourages the concurrent study of a broad range of academic areas. Students study: two modern languages (or a modern language and a classical language); a humanities or social science subject; an experimental science; mathematics; one of the creative arts. It is this comprehensive range of subjects that makes the Diploma Programme a demanding course of study designed to prepare students effectively for university entrance. In each of the academic areas students have flexibility in making their choices, which means they can choose subjects that particularly interest them and that they may wish to study further at university.

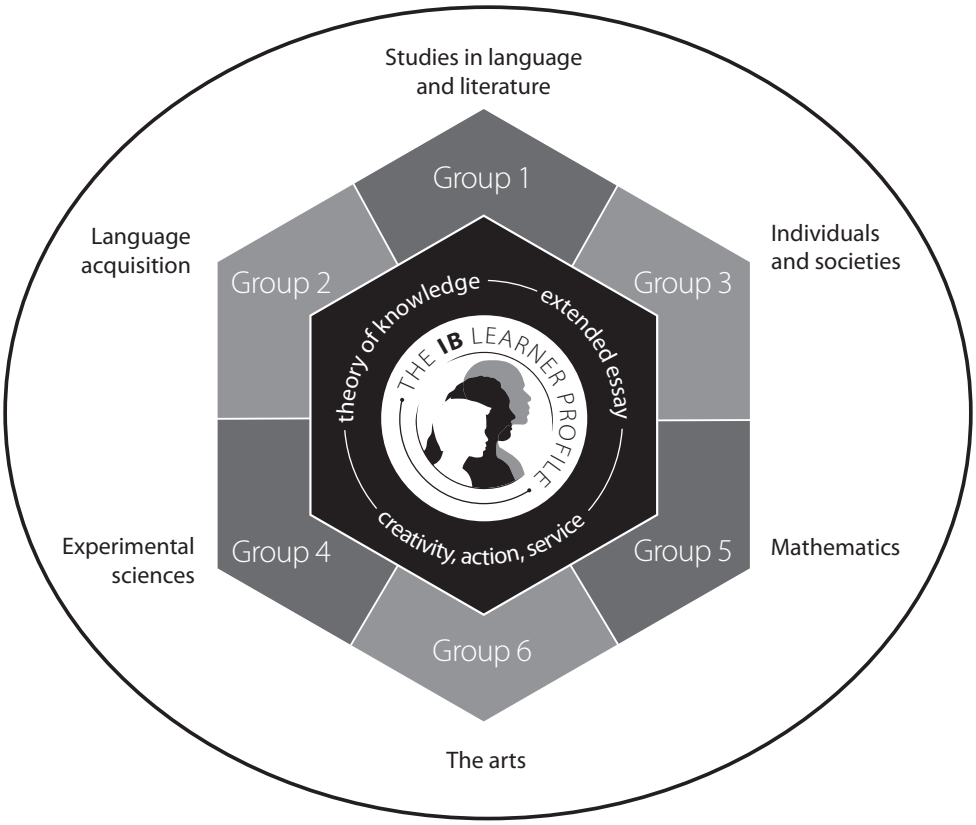

Figure 1
Diploma Programme model

Choosing the right combination

Students are required to choose one subject from each of the six academic areas, although they can choose a second subject from groups 1 to 5 instead of a group 6 subject. Normally, three subjects (and not more than four) are taken at higher level (HL), and the others are taken at standard level (SL). The IB recommends 240 teaching hours for HL subjects and 150 hours for SL. Subjects at HL are studied in greater depth and breadth than at SL.

At both levels, many skills are developed, especially those of critical thinking and analysis. At the end of the course, students' abilities are measured by means of external assessment. Many subjects contain some element of coursework assessed by teachers. The course is available for examinations in English, French and Spanish, with the exception of groups 1 and 2 courses where examinations are in the language of study.

The core of the hexagon

All Diploma Programme students participate in the three course requirements that make up the core of the hexagon. Reflection on all these activities is a principle that lies at the heart of the thinking behind the Diploma Programme.

The theory of knowledge course encourages students to think about the nature of knowledge, to reflect on the process of learning in all the subjects they study as part of their Diploma Programme course, and to make connections across the academic areas. The extended essay, a substantial piece of writing of up to 4,000 words, enables students to investigate a topic of special interest that they have chosen themselves. It also encourages them to develop the skills of independent research that will be expected at university. Creativity, action, service involves students in experiential learning through a range of artistic, sporting, physical and service activities.

The IB mission statement and the IB learner profile

The Diploma Programme aims to develop in students the knowledge, skills and attitudes they will need to fulfill the aims of the IB, as expressed in the organization's mission statement and the learner profile. Teaching and learning in the Diploma Programme represent the reality in daily practice of the organization's educational philosophy.

Nature of the subject

Introduction

The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: visual artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives, with all its interdisciplinary connections, provides a clear and sufficient rationale for making the study of this subject compulsory for students studying the full diploma.

Summary of courses available

Because individual students have different needs, interests and abilities, there are four different courses in mathematics. These courses are designed for different types of students: those who wish to study mathematics in depth, either as a subject in its own right or to pursue their interests in areas related to mathematics; those who wish to gain a degree of understanding and competence to understand better their approach to other subjects; and those who may not as yet be aware how mathematics may be relevant to their studies and in their daily lives. Each course is designed to meet the needs of a particular group of students. Therefore, great care should be taken to select the course that is most appropriate for an individual student.

In making this selection, individual students should be advised to take account of the following factors:

- their own abilities in mathematics and the type of mathematics in which they can be successful
- their own interest in mathematics and those particular areas of the subject that may hold the most interest for them
- their other choices of subjects within the framework of the Diploma Programme
- their academic plans, in particular the subjects they wish to study in future
- their choice of career.

Teachers are expected to assist with the selection process and to offer advice to students.

Mathematical studies SL

This course is available only at standard level, and is equivalent in status to mathematics SL, but addresses different needs. It has an emphasis on applications of mathematics, and the largest section is on statistical techniques. It is designed for students with varied mathematical backgrounds and abilities. It offers students

opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. The individual project is an extended piece of work based on personal research involving the collection, analysis and evaluation of data. Students taking this course are well prepared for a career in social sciences, humanities, languages or arts. These students may need to utilize the statistics and logical reasoning that they have learned as part of the mathematical studies SL course in their future studies.

Mathematics SL

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

Mathematics HL

This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems.

Further mathematics HL

This course is available only at higher level. It caters for students with a very strong background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students will expect to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications. It is expected that students taking this course will also be taking mathematics HL.

Note: Mathematics HL is an ideal course for students expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering or technology. It should not be regarded as necessary for such students to study further mathematics HL. Rather, further mathematics HL is an optional course for students with a particular aptitude and interest in mathematics, enabling them to study some wider and deeper aspects of mathematics, but is by no means a necessary qualification to study for a degree in mathematics.

Mathematical studies SL—course details

The course syllabus focuses on important mathematical topics that are interconnected. The syllabus is organized and structured with the following tenets in mind: placing more emphasis on student understanding of fundamental concepts than on symbolic manipulation and complex manipulative skills; giving greater emphasis to developing students' mathematical reasoning rather than performing routine operations; solving mathematical problems embedded in a wide range of contexts; using the calculator effectively.

The course includes project work, a feature unique to mathematical studies SL within group 5. Each student completes a project, based on their own research; this is guided and supervised by the teacher. The project provides an opportunity for students to carry out a mathematical study of their choice using their own experience, knowledge and skills acquired during the course. This process allows students to take sole responsibility for a part of their studies in mathematics.

The students most likely to select this course are those whose main interests lie outside the field of mathematics, and for many students this course will be their final experience of being taught formal mathematics. All parts of the syllabus have therefore been carefully selected to ensure that an approach starting from first principles can be used. As a consequence, students can use their own inherent, logical thinking skills and do not need to rely on standard algorithms and remembered formulae. Students likely to need mathematics for the achievement of further qualifications should be advised to consider an alternative mathematics course.

Owing to the nature of mathematical studies SL, teachers may find that traditional methods of teaching are inappropriate and that less formal, shared learning techniques can be more stimulating and rewarding for students. Lessons that use an inquiry-based approach, starting with practical investigations where possible, followed by analysis of results, leading to the understanding of a mathematical principle and its formulation into mathematical language, are often most successful in engaging the interest of students. Furthermore, this type of approach is likely to assist students in their understanding of mathematics by providing a meaningful context and by leading them to understand more fully how to structure their work for the project.

Prior learning

Mathematics is a linear subject, and it is expected that most students embarking on a Diploma Programme (DP) mathematics course will have studied mathematics for at least 10 years. There will be a great variety of topics studied, and differing approaches to teaching and learning. Thus, students will have a wide variety of skills and knowledge when they start the mathematical studies SL course. Most will have some background in arithmetic, algebra, geometry, trigonometry, probability and statistics. Some will be familiar with an inquiry approach, and may have had an opportunity to complete an extended piece of work in mathematics.

At the beginning of the syllabus section there is a list of topics that are considered to be prior learning for the mathematical studies SL course. It is recognized that this may contain topics that are unfamiliar to some students, but it is anticipated that there may be other topics in the syllabus itself that these students have already encountered. Teachers should plan their teaching to incorporate topics mentioned that are unfamiliar to their students.

Links to the Middle Years Programme

The prior learning topics for the DP courses have been written in conjunction with the Middle Years Programme (MYP) mathematics guide. The approaches to teaching and learning for DP mathematics build on the approaches used in the MYP. These include investigations, exploration and a variety of different assessment tools.

A continuum document called *Mathematics: The MYP–DP continuum* (November 2010) is available on the DP mathematics home pages of the online curriculum centre (OCC). This extensive publication focuses on the alignment of mathematics across the MYP and the DP. It was developed in response to feedback provided by IB World Schools, which expressed the need to articulate the transition of mathematics from the MYP to the DP. The publication also highlights the similarities and differences between MYP and DP mathematics, and is a valuable resource for teachers.

Mathematics and theory of knowledge

The *Theory of knowledge guide* (March 2006) identifies four ways of knowing, and it could be claimed that these all have some role in the acquisition of mathematical knowledge. While perhaps initially inspired by data from sense perception, mathematics is dominated by reason, and some mathematicians argue that their subject

is a language, that it is, in some sense, universal. However, there is also no doubt that mathematicians perceive beauty in mathematics, and that emotion can be a strong driver in the search for mathematical knowledge.

As an area of knowledge, mathematics seems to supply a certainty perhaps missing in other disciplines. This may be related to the "purity" of the subject that makes it sometimes seem divorced from reality. However, mathematics has also provided important knowledge about the world, and the use of mathematics in science and technology has been one of the driving forces for scientific advances.

Despite all its undoubted power for understanding and change, mathematics is in the end a puzzling phenomenon. A fundamental question for all knowers is whether mathematical knowledge really exists independently of our thinking about it. Is it there "waiting to be discovered" or is it a human creation?

Students' attention should be drawn to questions relating theory of knowledge (TOK) and mathematics, and they should be encouraged to raise such questions themselves, in mathematics and TOK classes. This includes questioning all the claims made above! Examples of issues relating to TOK are given in the "Links" column of the syllabus. Teachers could also discuss questions such as those raised in the "Areas of knowledge" section of the *Theory of knowledge guide*.

Mathematics and the international dimension

Mathematics is in a sense an international language, and, apart from slightly differing notation, mathematicians from around the world can communicate within their field. Mathematics transcends politics, religion and nationality, yet throughout history great civilizations owe their success in part to their mathematicians being able to create and maintain complex social and architectural structures.

Despite recent advances in the development of information and communication technologies, the global exchange of mathematical information and ideas is not a new phenomenon and has been essential to the progress of mathematics. Indeed, many of the foundations of modern mathematics were laid many centuries ago by Arabic, Greek, Indian and Chinese civilizations, among others. Teachers could use timeline websites to show the contributions of different civilizations to mathematics, but not just for their mathematical content. Illustrating the characters and personalities of the mathematicians concerned and the historical context in which they worked brings home the human and cultural dimension of mathematics.

The importance of science and technology in the everyday world is clear, but the vital role of mathematics is not so well recognized. It is the language of science, and underpins most developments in science and technology. A good example of this is the digital revolution, which is transforming the world, as it is all based on the binary number system in mathematics.

Many international bodies now exist to promote mathematics. Students are encouraged to access the extensive websites of international mathematical organizations to enhance their appreciation of the international dimension and to engage in the global issues surrounding the subject.

Examples of global issues relating to international-mindedness (**Int**) are given in the "Links" column of the syllabus.

Aims

Group 5 aims

The aims of all mathematics courses in group 5 are to enable students to:

1. enjoy mathematics, and develop an appreciation of the elegance and power of mathematics
2. develop an understanding of the principles and nature of mathematics
3. communicate clearly and confidently in a variety of contexts
4. develop logical, critical and creative thinking, and patience and persistence in problem-solving
5. employ and refine their powers of abstraction and generalization
6. apply and transfer skills to alternative situations, to other areas of knowledge and to future developments
7. appreciate how developments in technology and mathematics have influenced each other
8. appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
9. appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course.

Assessment objectives

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematical studies SL course, students will be expected to demonstrate the following.

1. **Knowledge and understanding**: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. **Problem-solving**: recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.
3. **Communication and interpretation**: transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation.
4. **Technology**: use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. **Reasoning**: construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.
6. **Investigative approaches**: investigate unfamiliar situations involving organizing and analysing information or measurements, drawing conclusions, testing their validity, and considering their scope and limitations.

Syllabus outline

Syllabus component	Teaching hours SL
All topics are compulsory. Students must study all the sub-topics in each of the topics in the syllabus as listed in this guide. Students are also required to be familiar with the topics listed as prior learning.	
Topic 1 Number and algebra	20
Topic 2 Descriptive statistics	12
Topic 3 Logic, sets and probability	20
Topic 4 Statistical applications	17
Topic 5 Geometry and trigonometry	18
Topic 6 Mathematical models	20
Topic 7 Introduction to differential calculus	18
Project The project is an individual piece of work involving the collection of information or the generation of measurements, and the analysis and evaluation of the information or measurements.	25
Total teaching hours	**150**

It is essential that teachers are allowed the prescribed minimum number of teaching hours necessary to meet the requirements of the mathematical studies SL course. At SL the minimum prescribed number of hours is 150 hours.

Approaches to the teaching and learning of mathematical studies SL

In this course the students will have the opportunity to understand and appreciate both the practical use of mathematics and its aesthetic aspects. They will be encouraged to build on knowledge from prior learning in mathematics and other subjects, as well as their own experience. It is important that students develop mathematical intuition and understand how they can apply mathematics in life.

Teaching needs to be flexible and to allow for different styles of learning. There is a diverse range of students in a mathematical studies SL classroom, and visual, auditory and kinaesthetic approaches to teaching may give new insights. The use of technology, particularly the graphic display calculator (GDC) and computer packages, can be very useful in allowing students to explore ideas in a rich context. It is left to the individual teacher to decide the order in which the separate topics are presented, but teaching and learning activities should weave the parts of the syllabus together and focus on their interrelationships. For example, the connection between geometric sequences and exponential functions can be illustrated by the consideration of compound interest.

Teachers may wish to introduce some topics using hand calculations to give an initial insight into the principles. However, once understanding has been gained, it is envisaged that the use of the GDC will support further work and simplify calculation (for example, the χ^2 statistic).

Teachers may take advantage of students' mathematical intuition by approaching the teaching of probability in a way that does not solely rely on formulae.

The mathematical studies SL project is meant to be not only an assessment tool, but also a sophisticated learning opportunity. It is an independent but well-guided piece of research, using mathematical methods to draw conclusions and answer questions from the individual student's interests. Project work should be incorporated into the course so that students are given the opportunity to learn the skills needed for the completion of a successful project. It is envisaged that the project will not be undertaken before students have experienced a range of techniques to make it meaningful. The scheme of work should be designed with this in mind.

Teachers should encourage students to find links and applications to their other IB subjects and the core of the hexagon. Everyday problems and questions should be drawn into the lessons to motivate students and keep the material relevant; suggestions are provided in the "Links" column of the syllabus.

For further information on "Approaches to teaching a DP course" please refer to the publication *The Diploma Programme: From principles into practice* (April 2009). To support teachers, a variety of resources can be found on the OCC and details of workshops for professional development are available on the public website.

Format of the syllabus

- **Content**: this column lists, under each topic, the sub-topics to be covered.
- **Further guidance**: this column contains more detailed information on specific sub-topics listed in the content column. This clarifies the content for examinations.

- **Links**: this column provides useful links to the aims of the mathematical studies SL course, with suggestions for discussion, real-life examples and project ideas. **These suggestions are only a guide for introducing and illustrating the sub-topic and are not exhaustive.** Links are labelled as follows.

  Appl real-life examples and links to other DP subjects

  Aim 8 moral, social and ethical implications of the sub-topic

  Int international-mindedness

  TOK suggestions for discussion

Note that any syllabus references to other subject guides given in the "Links" column are correct for the current (2012) published versions of the guides.

Course of study

The content of all seven topics in the syllabus must be taught, although not necessarily in the order in which they appear in this guide. Teachers are expected to construct a course of study that addresses the needs of their students and includes, where necessary, the topics noted in prior learning.

Integration of project work

Work leading to the completion of the project must be integrated into the course of study. Details of how to do this are given in the section on internal assessment and in the teacher support material.

Time allocation

The recommended teaching time for standard level courses is 150 hours. For mathematical studies SL, it is expected that 25 hours will be spent on work for the project. The time allocations given in this guide are approximate, and are intended to suggest how the remaining 125 hours allowed for the teaching of the syllabus might be allocated. However, the exact time spent on each topic depends on a number of factors, including the background knowledge and level of preparedness of each student. Teachers should therefore adjust these timings to correspond to the needs of their students.

Time has been allocated in each section of the syllabus to allow for the teaching of topics requiring the use of a GDC.

Use of calculators

Students are expected to have access to a GDC at all times during the course. The minimum requirements are reviewed as technology advances, and updated information will be provided to schools. It is expected that teachers and schools monitor calculator use with reference to the calculator policy. Regulations covering the types of calculators allowed in examinations are provided in the *Handbook of procedures for the Diploma Programme*. Further information and advice is provided in the *Mathematical studies SL: Graphic display calculators teacher support material* (May 2005) and on the OCC.

Mathematical studies SL formula booklet

Each student is required to have access to a clean copy of this booklet during the examination. It is recommended that teachers ensure students are familiar with the contents of this document from the beginning of the course. It is the responsibility of the school to download a copy from IBIS or the OCC, check that there are no printing errors, and ensure that there are sufficient copies available for all students.

Teacher support materials

A variety of teacher support materials will accompany this guide. These materials will include guidance for teachers on the introduction, planning and marking of projects, and specimen examination papers and markschemes.

Command terms and notation list

Teachers and students need to be familiar with the IB notation and the command terms, as these will be used without explanation in the examination papers. The “Glossary of command terms” and “Notation list” appear as appendices in this guide.

Prior learning topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematical studies SL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematical studies SL.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

The reference given in the left-hand column is to the topic in the syllabus content; for example, **1.0** refers to the prior learning for Topic 1—Number and algebra.

Learning how to use the graphic display calculator (GDC) effectively will be an integral part of the course, not a separate topic. Time has been allowed in each topic of the syllabus to do this.

	Content	Further guidance
1.0	Basic use of the four operations of arithmetic, using integers, decimals and fractions, including order of operations.	*Examples*: $2(3+4\times 7)=62$; $2\times 3+4\times 7=34$.
	Prime numbers, factors and multiples.	
	Simple applications of ratio, percentage and proportion.	
	Basic manipulation of simple algebraic expressions, including factorization and expansion.	*Examples*: $ab+ac=a(b+c)$; $(x+1)(x+2)=x^2+3x+2$.
	Rearranging formulae.	*Example*: $A=\frac{1}{2}bh \quad \Rightarrow \quad h=\frac{2A}{b}$.
	Evaluating expressions by substitution.	*Example*: If $x=-3$ then $x^2-2x+3=(-3)^2-2(-3)+3=18$.
	Solving linear equations in one variable.	*Examples*: $3(x+6)-4(x-1)=0$; $\frac{6x}{5}+4=7$.
	Solving systems of linear equations in two variables.	*Example*: $3x+4y=13$, $\frac{1}{3}x-2y=-1$.
	Evaluating exponential expressions with integer values.	*Examples*: $a^b, b\in\mathbb{Z}$; $2^{-4}=\frac{1}{16}$; $(-2)^4=16$.
	Use of inequalities $<, \leq, >, \geq$. Intervals on the real number line.	*Example*: $2<x\leq 5,\ x\in\mathbb{R}$.
	Solving linear inequalities.	*Example:* $2x+5<7-x$.
	Familiarity with commonly accepted world currencies.	*Examples*: Swiss franc (CHF); United States dollar (USD); British pound sterling (GBP); euro (EUR); Japanese yen (JPY); Australian dollar (AUD).

	Content	Further guidance
2.0	The collection of data and its representation in bar charts, pie charts and pictograms.	
5.0	Basic geometric concepts: point, line, plane, angle. Simple two-dimensional shapes and their properties, including perimeters and areas of circles, triangles, quadrilaterals and compound shapes. SI units for length and area. Pythagoras' theorem. Coordinates in two dimensions. Midpoints, distance between points.	

Syllabus content

Topic 1—Number and algebra 20 hours

The aims of this topic are to introduce some basic elements and concepts of mathematics, and to link these to financial and other applications.

	Content	Further guidance	Links
1.1	Natural numbers, $\mathbb{N}$; integers, $\mathbb{Z}$; rational numbers, $\mathbb{Q}$; and real numbers, $\mathbb{R}$. **Not required:** proof of irrationality, for example, of $\sqrt{2}$.	Link with domain and range 6.1.	**Int:** Historical development of number system. Awareness that our modern numerals are developed from the Arabic notation. **TOK:** Do mathematical symbols have sense in the same way that words have sense? Is zero different? Are these numbers created or discovered? Do these numbers exist?
1.2	Approximation: decimal places, significant figures. Percentage errors. Estimation.	Students should be aware of the errors that can result from premature rounding. Students should be able to recognize whether the results of calculations are reasonable, including reasonable values of, for example, lengths, angles and areas. For example, lengths cannot be negative.	**Appl:** Currency approximations to nearest whole number, eg peso, yen. Currency approximations to nearest cent/penny, eg euro, dollar, pound. **Appl:** Physics 1.1 (range of magnitudes). **Appl:** Meteorology, alternative rounding methods. **Appl:** Biology 2.1.5 (microscopic measurement). **TOK:** Appreciation of the differences of scale in number, and of the way numbers are used that are well beyond our everyday experience.

	Content	Further guidance	Links
1.3	Expressing numbers in the form $a \times 10^k$, where $1 \le a < 10$ and k is an integer. Operations with numbers in this form.	Students should be able to use scientific mode on the GDC. Calculator notation is *not* acceptable. For example, 5.2E3 is *not* acceptable.	**Appl:** Very large and very small numbers, eg astronomical distances, sub-atomic particles; Physics 1.1; global financial figures. **Appl:** Chemistry 1.1 (Avogadro's number). **Appl:** Physics 1.2 (scientific notation). **Appl:** Chemistry and biology (scientific notation). **Appl:** Earth science (earthquake measurement scale).
1.4	SI (*Système International*) and other basic units of measurement: for example, kilogram (kg), metre (m), second (s), litre (l), metre per second (m s^{-1}), Celsius scale.	Students should be able to convert between different units. Link with the form of the notation in 1.3, for example, $5\,\text{km} = 5 \times 10^6\,\text{mm}$.	**Appl:** Speed, acceleration, force; Physics 2.1, Physics 2.2; concentration of a solution; Chemistry 1.5. **Int:** SI notation. **TOK:** Does the use of SI notation help us to think of mathematics as a "universal language"? **TOK:** What is measurable? How can one measure mathematical ability?
1.5	Currency conversions.	Students should be able to perform currency transactions involving commission.	**Appl:** Economics 3.2 (exchange rates). **Aim 8:** The ethical implications of trading in currency and its effect on different national communities. **Int:** The effect of fluctuations in currency rates on international trade.

	Content	Further guidance	Links
1.6	Use of a GDC to solve • pairs of linear equations in two variables • quadratic equations.	In examinations, no specific method of solution will be required. Standard terminology, such as zeros or roots, should be taught. Link with quadratic models in 6.3.	**TOK:** Equations with no solutions. Awareness that when mathematicians talk about "imaginary" or "real" solutions they are using precise technical terms that do not have the same meaning as the everyday terms.
1.7	Arithmetic sequences and series, and their applications. Use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence.	Students may use a GDC for calculations, but they will be expected to identify the first term and the common difference.	**TOK:** Informal and formal reasoning in mathematics. How does mathematical proof differ from good reasoning in everyday life? Is mathematical reasoning different from scientific reasoning? **TOK:** Beauty and elegance in mathematics. Fibonacci numbers and connections with the Golden ratio.
1.8	Geometric sequences and series. Use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence. **Not required:** formal proofs of formulae. **Not required:** use of logarithms to find *n*, given the sum of the first *n* terms; sums to infinity.	Students may use a GDC for calculations, but they will be expected to identify the first term and the common ratio.	

	Content	Further guidance	Links
1.9	Financial applications of geometric sequences and series: • compound interest • annual depreciation. **Not required:** use of logarithms.	Use of the GDC is expected, including built-in financial packages. The concept of simple interest may be used as an introduction to compound interest but will not be examined. In examinations, questions that ask students to derive the formula will not be set. Compound interest can be calculated yearly, half-yearly, quarterly or monthly. Link with exponential models 6.4.	**Appl:** Economics 3.2 (exchange rates). **Aim 8:** Ethical perceptions of borrowing and lending money. **Int:** Do all societies view investment and interest in the same way?

Topic 2—Descriptive statistics 12 hours

The aim of this topic is to develop techniques to describe and interpret sets of data, in preparation for further statistical applications.

	Content	Further guidance	Links
2.1	Classification of data as discrete or continuous.	Students should understand the concept of population and of representative and random sampling. Sampling will not be examined but can be used in internal assessment.	**Appl:** Psychology 3 (research methodology). **Appl:** Biology 1 (statistical analysis). **TOK:** Validity of data and introduction of bias.
2.2	Simple discrete data: frequency tables.		
2.3	Grouped discrete or continuous data: frequency tables; mid-interval values; upper and lower boundaries. Frequency histograms.	In examinations, frequency histograms will have equal class intervals.	**Appl:** Geography (geographical analyses).
2.4	Cumulative frequency tables for grouped discrete data and for grouped continuous data; cumulative frequency curves, median and quartiles. Box-and-whisker diagram. **Not required:** treatment of outliers.	Use of GDC to produce histograms and box-and-whisker diagrams.	
2.5	Measures of central tendency. For simple discrete data: mean; median; mode. For grouped discrete and continuous data: estimate of a mean; modal class.	Students should use mid-interval values to estimate the mean of grouped data. In examinations, questions using $\sum$ notation will not be set.	**Aim 8:** The ethical implications of using statistics to mislead.

	Content	Further guidance	Links
2.6	Measures of dispersion: range, interquartile range, standard deviation.	Students should use mid-interval values to estimate the standard deviation of grouped data. In examinations: • students are expected to use a GDC to calculate standard deviations • the data set will be treated as the population. Students should be aware that the IB notation may differ from the notation on their GDC. Use of computer spreadsheet software is encouraged in the treatment of this topic.	**Int:** The benefits of sharing and analysing data from different countries. **TOK:** Is standard deviation a mathematical discovery or a creation of the human mind?

Topic 3—Logic, sets and probability 20 hours

The aims of this topic are to introduce the principles of logic, to use set theory to introduce probability, and to determine the likelihood of random events using a variety of techniques.

	Content	Further guidance	Links
3.1	Basic concepts of symbolic logic: definition of a proposition; symbolic notation of propositions.		
3.2	Compound statements: implication, $\Rightarrow$; equivalence, $\Leftrightarrow$; negation, $\neg$; conjunction, $\wedge$; disjunction, $\vee$; exclusive disjunction, $\veebar$. Translation between verbal statements and symbolic form.		
3.3	Truth tables: concepts of logical contradiction and tautology.	A maximum of three propositions will be used in truth tables. Truth tables can be used to illustrate the associative and distributive properties of connectives, and for variations of implication and equivalence statements, for example, $\neg q \Rightarrow \neg p$.	
3.4	Converse, inverse, contrapositive. Logical equivalence. Testing the validity of simple arguments through the use of truth tables.	The topic may be extended to include syllogisms. In examinations these will not be tested.	**Appl:** Use of arguments in developing a logical essay structure. **Appl:** Computer programming; digital circuits; Physics HL 14.1; Physics SL C1. **TOK:** Inductive and deductive logic, fallacies.

	Content	Further guidance	Links
3.5	Basic concepts of set theory: elements $x \in A$, subsets $A \subset B$; intersection $A \cap B$; union $A \cup B$; complement A'. Venn diagrams and simple applications. **Not required:** knowledge of de Morgan's laws.	In examinations, the universal set U will include no more than three subsets. The empty set is denoted by $\varnothing$.	
3.6	Sample space; event A; complementary event, A'. Probability of an event. Probability of a complementary event. Expected value.	Probability may be introduced and taught in a practical way using coins, dice, playing cards and other examples to demonstrate random behaviour. In examinations, questions involving playing cards will not be set.	**Appl:** Actuarial studies, probability of life spans and their effect on insurance. **Appl:** Government planning based on projected figures. **TOK:** Theoretical and experimental probability.
3.7	Probability of combined events, mutually exclusive events, independent events. Use of tree diagrams, Venn diagrams, sample space diagrams and tables of outcomes. Probability using "with replacement" and "without replacement". Conditional probability.	Students should be encouraged to use the most appropriate method in solving individual questions. Probability questions will be placed in context and will make use of diagrammatic representations. In examinations, questions requiring the exclusive use of the formula in section 3.7 of the formula booklet will not be set.	**Appl:** Biology 4.3 (theoretical genetics); Biology 4.3.2 (Punnett squares). **Appl:** Physics HL13.1 (determining the position of an electron); Physics SL B1. **Aim 8:** The ethics of gambling. **TOK:** The perception of risk, in business, in medicine and safety in travel.

Topic 4—Statistical applications 17 hours

The aims of this topic are to develop techniques in inferential statistics in order to analyse sets of data, draw conclusions and interpret these.

	Content	Further guidance	Links
4.1	The normal distribution. The concept of a random variable; of the parameters μ and σ; of the bell shape; the symmetry about $x = \mu$.	Students should be aware that approximately 68% of the data lies between $\mu \pm \sigma$, 95% lies between $\mu \pm 2\sigma$ and 99% lies between $\mu \pm 3\sigma$.	**Appl:** Examples of measurements, ranging from psychological to physical phenomena, that can be approximated, to varying degrees, by the normal distribution. **Appl:** Biology 1 (statistical analysis). **Appl:** Physics 3.2 (kinetic molecular theory).
	Diagrammatic representation.	Use of sketches of normal curves and shading when using the GDC is expected.	
	Normal probability calculations.	Students will be expected to use the GDC when calculating probabilities and inverse normal.	
	Expected value.		
	Inverse normal calculations.	In examinations, inverse normal questions will not involve finding the mean or standard deviation.	
	Not required: Transformation of any normal variable to the standardized normal.	Transformation of any normal variable to the standardized normal variable, z, may be appropriate in internal assessment. In examinations, questions requiring the use of z scores will not be set.	

	Content	Further guidance	Links
4.2	Bivariate data: the concept of correlation. Scatter diagrams; line of best fit, by eye, passing through the mean point. Pearson's product–moment correlation coefficient, r. Interpretation of positive, zero and negative, strong or weak correlations.	Students should be able to make the distinction between correlation and causation. Hand calculations of r may enhance understanding. In examinations, students will be expected to use a GDC to calculate r.	**Appl:** Biology; Physics; Chemistry; Social sciences. **TOK:** Does correlation imply causation?
4.3	The regression line for y on x. Use of the regression line for prediction purposes.	Hand calculations of the regression line may enhance understanding. In examinations, students will be expected to use a GDC to find the regression line. Students should be aware of the dangers of extrapolation.	**Appl:** Chemistry 11.3 (graphical techniques). **TOK:** Can we reliably use the equation of the regression line to make predictions?

	Content	Further guidance	Links
4.4	The χ^2 test for independence: formulation of null and alternative hypotheses; significance levels; contingency tables; expected frequencies; degrees of freedom; *p*-values.	In examinations: • the maximum number of rows or columns in a contingency table will be 4 • the degrees of freedom will always be greater than one • the χ^2 critical value will always be given • only questions on upper tail tests with commonly used significance levels (1%, 5%, 10%) will be set. Calculation of expected frequencies by hand is required. Hand calculations of χ^2 may enhance understanding. In examinations students will be expected to use the GDC to calculate the χ^2 statistic. If using χ^2 tests in internal assessment, students should be aware of the limitations of the test for small expected frequencies; **expected frequencies** must be greater than 5. If the degree of freedom is 1, then Yates's continuity correction should be applied.	**Appl:** Biology (internal assessment); Psychology; Geography. **TOK:** Scientific method.

Topic 5—Geometry and trigonometry 18 hours

The aims of this topic are to develop the ability to draw clear diagrams in two dimensions, and to apply appropriate geometric and trigonometric techniques to problem-solving in two and three dimensions.

	Content	Further guidance	Links
5.1	Equation of a line in two dimensions: the forms $y = mx + c$ and $ax + by + d = 0$. Gradient; intercepts. Points of intersection of lines. Lines with gradients, m_1 and m_2. Parallel lines $m_1 = m_2$. Perpendicular lines, $m_1 \times m_2 = -1$.	Link with linear functions in 6.2. Link with solutions of pairs of linear equations in 1.6.	**Appl:** Gradients of mountain roads, eg Canadian Highway. Gradients of access ramps. **Appl:** Economics 1.2 (elasticity). **TOK:** Descartes showed that geometric problems can be solved algebraically and vice versa. What does this tell us about mathematical representation and mathematical knowledge?
5.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. Angles of elevation and depression.	Problems may incorporate Pythagoras' theorem. In examinations, questions will only be set in degrees.	**Appl:** Triangulation, map-making, finding practical measurements using trigonometry. **Int:** Diagrams of Pythagoras' theorem occur in early Chinese and Indian manuscripts. The earliest references to trigonometry are in Indian mathematics.

	Content	Further guidance	Links
5.3	Use of the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.	In all areas of this topic, students should be encouraged to sketch well-labelled diagrams to support their solutions. The ambiguous case could be taught, but will not be examined. In examinations, questions will only be set in degrees.	**Appl:** Vectors; Physics 1.3; bearings.
	Use of the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$; $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$. Use of area of a triangle $= \frac{1}{2}ab\sin C$. Construction of labelled diagrams from verbal statements.		**TOK:** Use the fact that the cosine rule is one possible generalization of Pythagoras' theorem to explore the concept of "generality".

	Content	Further guidance	Links
5.4	Geometry of three-dimensional solids: cuboid; right prism; right pyramid; right cone; cylinder; sphere; hemisphere; and combinations of these solids. The distance between two points; eg between two vertices or vertices with midpoints or midpoints with midpoints. The size of an angle between two lines or between a line and a plane. **Not required:** angle between two planes.	In examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes.	**TOK:** What is an axiomatic system? Do the angles in a triangle always add to 180°? Non-Euclidean geometry, such as Riemann's. Flight maps of airlines. **Appl:** Architecture and design.
5.5	Volume and surface areas of the three-dimensional solids defined in 5.4.		

Topic 6—Mathematical models 20 hours

The aim of this topic is to develop understanding of some mathematical functions that can be used to model practical situations. Extensive use of a GDC is to be encouraged in this topic.

	Content	Further guidance	Links
6.1	Concept of a function, domain, range and graph. Function notation, eg $f(x)$, $v(t)$, $C(n)$. Concept of a function as a mathematical model.	In examinations: • the domain is the set of all real numbers unless otherwise stated • mapping notation $f: x \mapsto y$ will not be used.	**TOK:** Why can we use mathematics to describe the world and make predictions? Is it because we discover the mathematical basis of the world or because we impose our own mathematical structures onto the world? The relationship between real-world problems and mathematical models.
6.2	Linear models. Linear functions and their graphs, $f(x) = mx + c$.	Link with equation of a line in 5.1.	**Appl:** Conversion graphs, eg temperature or currency conversion; Physics 3.1; Economics 3.2.
6.3	Quadratic models. Quadratic functions and their graphs (parabolas): $f(x) = ax^2 + bx + c$; $a \neq 0$ Properties of a parabola: symmetry; vertex; intercepts on the x-axis and y-axis. Equation of the axis of symmetry, $x = -\frac{b}{2a}$.	Link with the quadratic equations in 1.6. Functions with zero, one or two real roots are included. The form of the equation of the axis of symmetry may initially be found by investigation. Properties should be illustrated with a GDC or graphical software.	**Appl:** Cost functions; projectile motion; Physics 9.1; area functions.

	Content	Further guidance	Links
6.4	Exponential models. Exponential functions and their graphs: $f(x) = ka^x + c;\ a \in \mathbb{Q}^+, a \neq 1, k \neq 0$. $f(x) = ka^{-x} + c;\ a \in \mathbb{Q}^+, a \neq 1, k \neq 0$. Concept and equation of a horizontal asymptote.	In examinations, students will be expected to use graphical methods, including GDCs, to solve problems.	**Appl:** Biology 5.3 (populations). **Appl:** Biology 5.3.2 (population growth); Physics 13.2 (radioactive decay); Physics I2 (X-ray attenuation); cooling of a liquid; spread of a virus; depreciation.
6.5	Models using functions of the form $f(x) = ax^m + bx^n + \ldots;\ m, n \in \mathbb{Z}$. Functions of this type and their graphs. The y-axis as a vertical asymptote.	In examinations, students will be expected to use graphical methods, including GDCs, to solve problems. *Examples*: $f(x) = 3x^4 - 5x + 3$, $g(x) = 3x^2 - \frac{4}{x}$.	
6.6	Drawing accurate graphs. Creating a sketch from information given. Transferring a graph from GDC to paper. Reading, interpreting and making predictions using graphs. Included all the functions above and additions and subtractions.	Students should be aware of the difference between the command terms "draw" and "sketch". All graphs should be labelled and have some indication of scale. *Examples*: $f(x) = x^3 + 5 - \frac{2}{x}$, $g(x) = 3^{-x} + x$.	**TOK:** Does a graph without labels or indication of scale have meaning?

	Content	Further guidance	Links
6.7	Use of a GDC to solve equations involving combinations of the functions above.	*Examples*: $x+2=2x^3+3x-1$, $5x=3^x$. Other functions can be used for modelling in internal assessment but will not be set on examination papers.	

Topic 7—Introduction to differential calculus 18 hours

The aim of this topic is to introduce the concept of the derivative of a function and to apply it to optimization and other problems.

	Content	Further guidance	Links
7.1	Concept of the derivative as a rate of change. Tangent to a curve. **Not required:** formal treatment of limits.	Teachers are encouraged to introduce differentiation through a graphical approach, rather than a formal treatment. Emphasis is placed on interpretation of the concept in different contexts. In examinations, questions on differentiation from first principles will not be set.	**Appl:** Rates of change in economics, kinematics and medicine. **Aim 8:** Plagiarism and acknowledgment of sources, eg the conflict between Newton and Leibnitz, who approached the development of calculus from different directions **TOK:** Is intuition a valid way of knowing in maths? How is it possible to reach the same conclusion from different research paths?
7.2	The principle that $f(x) = ax^n \quad \Rightarrow \quad f'(x) = anx^{n-1}$. The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$, where all exponents are integers.	Students should be familiar with the alternative notation for derivatives $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}V}{\mathrm{d}r}$. In examinations, knowledge of the second derivative will not be assumed.	

	Content	Further guidance	Links
7.3	Gradients of curves for given values of x. Values of x where $f'(x)$ is given. Equation of the tangent at a given point. Equation of the line perpendicular to the tangent at a given point (normal).	The use of technology to find the gradient at a point is also encouraged. The use of technology to draw tangent and normal lines is also encouraged. Links with perpendicular lines in 5.1.	
7.4	Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$.		
7.5	Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$. Stationary points. Local maximum and minimum points.	The use of technology to display $f(x)$ and $f'(x)$, and find the solutions of $f'(x) = 0$ is also encouraged. Awareness that a local maximum/minimum will not necessarily be the greatest/least value of the function in the given domain. Awareness of points of inflexion with zero gradient is to be encouraged, but will not be examined.	
7.6	Optimization problems.	*Examples*: Maximizing profit, minimizing cost, maximizing volume for given surface area. In examinations, questions on kinematics will not be set.	**Appl:** Efficient use of material in packaging. **Appl:** Physics 2.1 (kinematics).

Assessment in the Diploma Programme

General

Assessment is an integral part of teaching and learning. The most important aims of assessment in the Diploma Programme are that it should support curricular goals and encourage appropriate student learning. Both external and internal assessment are used in the Diploma Programme. IB examiners mark work produced for external assessment, while work produced for internal assessment is marked by teachers and externally moderated by the IB.

There are two types of assessment identified by the IB.

- Formative assessment informs both teaching and learning. It is concerned with providing accurate and helpful feedback to students and teachers on the kind of learning taking place and the nature of students' strengths and weaknesses in order to help develop students' understanding and capabilities. Formative assessment can also help to improve teaching quality, as it can provide information to monitor progress towards meeting the course aims and objectives.
- Summative assessment gives an overview of previous learning and is concerned with measuring student achievement.

The Diploma Programme primarily focuses on summative assessment designed to record student achievement at or towards the end of the course of study. However, many of the assessment instruments can also be used formatively during the course of teaching and learning, and teachers are encouraged to do this. A comprehensive assessment plan is viewed as being integral with teaching, learning and course organization. For further information, see the IB *Programme standards and practices* document.

The approach to assessment used by the IB is criterion-related, not norm-referenced. This approach to assessment judges students' work by their performance in relation to identified levels of attainment, and not in relation to the work of other students. For further information on assessment within the Diploma Programme, please refer to the publication *Diploma Programme assessment: Principles and practice*.

To support teachers in the planning, delivery and assessment of the Diploma Programme courses, a variety of resources can be found on the OCC or purchased from the IB store (http://store.ibo.org). Teacher support materials, subject reports, internal assessment guidance, grade descriptors, as well as resources from other teachers, can be found on the OCC. Specimen and past examination papers, as well as markschemes, can be purchased from the IB store.

Methods of assessment

The IB uses several methods to assess work produced by students.

Assessment criteria

Assessment criteria are used when the assessment task is open-ended. Each criterion concentrates on a particular skill that students are expected to demonstrate. An assessment objective describes what students should be able to do, and assessment criteria describe how well they should be able to do it. Using assessment criteria allows discrimination between different answers and encourages a variety of responses. Each criterion

comprises a set of hierarchically ordered level descriptors. Each level descriptor is worth one or more marks. Each criterion is applied independently using a best-fit model. The maximum marks for each criterion may differ according to the criterion's importance. The marks awarded for each criterion are added together to give the total mark for the piece of work.

Markbands

Markbands are a comprehensive statement of expected performance against which responses are judged. They represent a single holistic criterion divided into level descriptors. Each level descriptor corresponds to a range of marks to differentiate student performance. A best-fit approach is used to ascertain which particular mark to use from the possible range for each level descriptor.

Markschemes

This generic term is used to describe analytic markschemes that are prepared for specific examination papers. Analytic markschemes are prepared for those examination questions that expect a particular kind of response and/or a given final answer from the students. They give detailed instructions to examiners on how to break down the total mark for each question for different parts of the response. A markscheme may include the content expected in the responses to questions or may be a series of marking notes giving guidance on how to apply criteria.

Assessment outline

First examinations 2014

Assessment component	Weighting
External assessment (3 hours)	**80%**
Paper 1 (1 hour 30 minutes) 15 compulsory short-response questions based on the whole syllabus. (90 marks)	**40%**
Paper 2 (1 hour 30 minutes) 6 compulsory extended-response questions based on the whole syllabus. (90 marks)	**40%**
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Project** The project is an individual piece of work involving the collection of information or the generation of measurements, and the analysis and evaluation of the information or measurements. (20 marks)	**20%**

External assessment

General

Markschemes are used to assess students in both papers. The markschemes are specific to each examination.

External assessment details

General information

Paper 1 and paper 2

These papers are externally set and externally marked. Together, they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Calculators

For both examination papers, students must have access to a GDC at all times. Regulations covering the types of GDC allowed are provided in the *Handbook of procedures for the Diploma Programme*.

Mathematical studies SL formula booklet

Each student must have access to a clean copy of the formula booklet during the examination. It is the responsibility of the school to download a copy from IBIS or the OCC and to ensure that there are sufficient copies available for all students.

Awarding of marks

In addition to correct answers, marks are awarded for method, accuracy and reasoning.

In paper 1, full marks are awarded for each correct answer irrespective of the presence or absence of working. Where an answer is incorrect, marks are given for correct method. All students should therefore be advised to show their working.

In paper 2, full marks are not necessarily awarded for a correct answer without working. Answers must be supported by working and/or explanations. Where an answer is incorrect, marks are given for correct method. All students should therefore be advised to show their working.

Paper 1

Duration: 1 hour 30 minutes
Weighting: 40%

- This paper consists of 15 compulsory short-response questions.
- Each question is worth 6 marks.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.
- The intention of this paper is to test students' knowledge and understanding across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Questions of varying levels of difficulty are set.
- One or more steps may be needed to answer each question.

Paper 2

Duration: 1 hour 30 minutes
Weighting: 40%

- This paper consists of 6 compulsory extended-response questions.
- Questions in this section will vary in terms of length and level of difficulty.
- Individual questions will not be worth the same number of marks. The marks allocated are indicated at the start of each question.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.
- The intention of this paper is to test students' knowledge and understanding of the syllabus in depth. The range of syllabus topics tested in this paper may be narrower than that tested in paper 1.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may require knowledge of more than one topic.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem-solving.

Internal assessment

Purpose of internal assessment

Internal assessment is an integral part of the course and is compulsory for all students. It enables students to demonstrate the application of their skills and knowledge, and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations. The internal assessment should, as far as possible, be woven into normal classroom teaching and not be a separate activity conducted after a course has been taught.

Internal assessment in mathematical studies SL is an individual project. This is a piece of written work based on personal research involving the collection, analysis and evaluation of data. It is marked according to seven assessment criteria.

Guidance and authenticity

The project submitted for internal assessment must be the student's own work. However, it is not the intention that students should decide upon a title or topic and be left to work on the project without any further support from the teacher. The teacher should play an important role during both the planning stage and the period when the student is working on the project. It is the responsibility of the teacher to ensure that students are familiar with:

- the requirements of the type of work to be internally assessed
- the IB academic honesty policy available on the OCC
- the assessment criteria—students must understand that the work submitted for assessment must address these criteria effectively.

Teachers and students must discuss the project. Students should be encouraged to initiate discussions with the teacher to obtain advice and information, and students must not be penalized for seeking guidance. However, if a student could not have completed the project without substantial support from the teacher, this should be recorded on the appropriate form from the *Handbook of procedures for the Diploma Programme*.

It is the responsibility of teachers to ensure that all students understand the basic meaning and significance of concepts that relate to academic honesty, especially authenticity and intellectual property. Teachers must ensure that all student work for assessment is prepared according to the requirements and must explain clearly to students that the project must be entirely their own.

As part of the learning process, teachers can give advice to students on a **first draft** of the project. This advice should be in terms of the way the work could be improved, but this first draft must not be heavily annotated or edited by the teacher. The next version handed to the teacher after the first draft must be the final one.

All work submitted to the IB for moderation or assessment must be authenticated by a teacher, and must not include any known instances of suspected or confirmed malpractice. Each student must sign the coversheet for internal assessment to confirm that the work is his or her authentic work and constitutes the final version of that work. Once a student has officially submitted the final version of the work to a teacher (or the coordinator) for internal assessment, together with the signed coversheet, it cannot be retracted.

Authenticity may be checked by discussion with the student on the content of the work, and scrutiny of one or more of the following:

- the student's initial proposal
- the first draft of the written work
- the references cited
- the style of writing compared with work known to be that of the student.

Authenticity must be verified by the signing of the relevant form from the *Handbook of Procedures for the Diploma Programme* by both student and teacher.

By supervising students throughout, teachers should be monitoring the progress individual students are making and be in a position to discuss with them the source of any new material that appears, or is referred to, in their projects. Often, students are not aware when it is permissible to use material written by others or when to seek help from other sources. Consequently, open discussion in the early stages is a good way of avoiding these potential problems.

However, if teachers are unsure as to whether a project is the student's own work they should employ a range of methods to check this fact. These may include:

- discussing with the student
- asking the student to explain the methods used and to summarize the results and conclusions
- asking the student to replicate part of the analysis using different data
- inviting the student to give a class presentation of his or her project

The requirement for teachers and students to sign the coversheet for internal assessment applies to the work of all students, not just the sample work that will be submitted to an examiner for the purpose of moderation. If the teacher and student sign a coversheet, but there is a comment to the effect that the work may not be authentic, the student will not be eligible for a mark in that component and no grade will be awarded. For further details refer to the IB publication *Academic honesty* and the relevant articles in the *General regulations: Diploma Programme*.

The same piece of work cannot be submitted to meet the requirements of both the internal assessment and the extended essay.

Group work

Group work should not be used for projects. Each project is an individual piece of work based on different data collected or measurements generated.

It should be made clear to students that all work connected with the project, including the writing of the project, should be their own. It is therefore helpful if teachers try to encourage in students a sense of responsibility for their own learning so that they accept a degree of ownership and take pride in their own work.

Time allocation

Internal assessment is an integral part of the mathematical studies SL course, contributing 20% to the final assessment in the course. This weighting should be reflected in the time that is allocated to teaching the knowledge, skills and understanding required to undertake the work as well as the total time allocated to carry out the work.

It is expected that a total of approximately 25 teaching hours should be allocated to the work. This should include:

- time for the teacher to explain to students the requirements of the project
- class time for students to work on the project
- time for consultation between the teacher and each student
- time to review and monitor progress, and to check authenticity.

Using assessment criteria for internal assessment

For internal assessment, a number of assessment criteria have been identified. Each assessment criterion has level descriptors describing specific levels of achievement together with an appropriate range of marks. The level descriptors concentrate on positive achievement, although for the lower levels failure to achieve may be included in the description.

Teachers must judge the internally assessed work against the criteria using the level descriptors.

- The aim is to find, for each criterion, the descriptor that conveys most accurately the level attained by the student.
- When assessing a student's work, teachers should read the level descriptors for each criterion, starting with level 0, until they reach a descriptor that describes a level of achievement that has not been reached. The level of achievement gained by the student is therefore the preceding one, and it is this that should be recorded.
- Only whole numbers should be recorded; partial marks, that is fractions and decimals, are not acceptable.
- Teachers should not think in terms of a pass or fail boundary, but should concentrate on identifying the appropriate descriptor for each assessment criterion.
- The highest level descriptors do not imply faultless performance but should be achievable by a student. Teachers should not hesitate to use the extremes if they are appropriate descriptions of the work being assessed.
- A student who attains a high level of achievement in relation to one criterion will not necessarily attain high levels of achievement in relation to the other criteria. Similarly, a student who attains a low level of achievement for one criterion will not necessarily attain low achievement levels for the other criteria. Teachers should not assume that the overall assessment of the students will produce any particular distribution of marks.
- It is expected that the assessment criteria be made available to students.

Internal assessment details

Project

Duration: 25 teaching hours
Weighting: 20%

The purpose of the project

The aims of the mathematical studies SL course are carried through into the objectives that are formally assessed as part of the course, either through written examination papers, or the project, or both. The assessment criteria for the project have been developed to address these objectives. In addition to formally

testing the objectives of the course, project work provides opportunities for students to achieve competence in areas that will contribute to their overall education, as well as to acquire qualities that are likely to contribute to their personal development.

The specific purposes of the project are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- encourage students to initiate and sustain a piece of work in mathematics
- enable students to acquire confidence in developing strategies for dealing with new situations and problems
- provide opportunities for students to develop individual skills and techniques, and to allow students with varying abilities, interests and experiences to achieve a sense of personal satisfaction in studying mathematics
- enable students to experience mathematics as an integrated organic discipline rather than fragmented and compartmentalized skills and knowledge
- enable students to see connections and applications of mathematics to other areas of interest
- provide opportunities for students to show, with confidence, what they know and what they can do.

Introduction of the project

Project work should be incorporated into the course so that students are given the opportunity to learn the skills needed for the completion of a successful project.

Time in class can therefore be used:

- for general discussion of areas of study for project work, such as: how data can be collected or measurements generated; where data can be collected; how much data should be collected; different ways of displaying data; what steps should be taken to analyse the data; how data should be evaluated
- to give students the opportunity to review and mark projects from previous years, using the assessment criteria.

Further details on the development of the project are included in the teacher support material.

Requirements and recommendations

Each project must contain:

- a title
- a statement of the task and plan
- measurements, information or data that have been collected and/or generated
- an analysis of the measurements, information or data
- interpretation of results, including a discussion of validity
- appropriate notation and terminology.

Historical projects that reiterate facts but have little mathematical content are not appropriate and should be actively discouraged.

Work set by the teacher is not appropriate for a project.

Students can choose from a wide variety of project types, for example, modelling, investigations, applications and statistical surveys.

The project should not normally exceed **2,000** words, excluding diagrams, graphs, appendices and bibliography. However, it is the quality of the mathematics and the processes used and described that is important, rather than the number of words written.

The teacher is expected to give appropriate guidance at all stages of the project by, for example, directing students into more productive routes of inquiry, making suggestions for suitable sources of information, and providing general advice on the content and clarity of a project in the writing-up stage.

Teachers are responsible for indicating to students the existence of errors but should not explicitly correct these errors. It must be emphasized that students are expected to consult the teacher throughout the process.

All students should be familiar with the requirements of the project and the criteria by which it is assessed. Students need to start planning their projects as early as possible in the course. Deadlines, preferably reached by agreement between students and teachers, need to be firmly established. There needs to be a date for submission of the project title and a brief outline description, a date for the completion of data collection or generation, a date for the submission of the first draft and, of course, a date for project completion.

In developing their projects, students should make use of mathematics learned as part of the course. The level of sophistication of the mathematics should be similar to that suggested by the syllabus. It is not expected that students produce work that is outside the mathematical studies SL syllabus—however, this is not penalized.

Internal assessment criteria

The project is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematical studies SL.

Each project is assessed against the following seven criteria. The final mark for each project is the sum of the scores for each criterion. The maximum possible final mark is 20.

Students will not receive a grade for mathematical studies SL if they have not submitted a project.

Criterion A	Introduction
Criterion B	Information/measurement
Criterion C	Mathematical processes
Criterion D	Interpretation of results
Criterion E	Validity
Criterion F	Structure and communication
Criterion G	Notation and terminology

Criterion A: Introduction

In this context, the word "task" is defined as "what the student is going to do"; the word "plan" is defined as "how the student is going to do it". A statement of the task should appear at the beginning of each project. It is expected that each project has a clear title.

Achievement level	Descriptor
0	The project does not contain a clear statement of the task. *There is no evidence in the project of any statement of what the student is going to do or has done.*
1	The project contains a clear statement of the task. *For this level to be achieved, the task should be stated explicitly.*
2	The project contains a title, a clear statement of the task and a description of the plan. *The plan need not be highly detailed, but must describe how the task will be performed. If the project does not have a title, this achievement level cannot be awarded.*
3	The project contains a title, a clear statement of the task and a detailed plan that is followed. *The plan should specify what techniques are to be used at each stage and the purpose behind them, thus lending focus to the task.*

Criterion B: Information/measurement

In this context, generated measurements include those that have been generated by computer, by observation, by prediction from a mathematical model or by experiment. Mathematical information includes geometrical figures and data that is collected empirically or assembled from outside sources. This list is not exhaustive and mathematical information does not solely imply data for statistical analysis. If a questionnaire or survey is used then a copy of this along with the raw data must be included.

Achievement level	Descriptor
0	The project does not contain any relevant information collected or relevant measurements generated. *No attempt has been made to collect any relevant information or to generate any relevant measurements.*
1	The project contains relevant information collected or relevant generated measurements. *This achievement level can be awarded even if a fundamental flaw exists in the instrument used to collect the information, for example, a faulty questionnaire or an interview conducted in an invalid way.*

Achievement level	Descriptor
2	The relevant information collected, or set of measurements generated, is organized in a form appropriate for analysis **or** is sufficient in both quality and quantity. *A satisfactory attempt has been made to structure the information/measurements ready for the process of analysis,* ***or*** *the information/measurement collection process has been thoroughly described and the quantity of information justified. The raw data must be included for this achievement level to be awarded.*
3	The relevant information collected, or set of measurements generated, is organized in a form appropriate for analysis **and** is sufficient in both quality and quantity. *The information/measurements have been properly structured ready for analysis* ***and*** *the information/measurement collection process has been thoroughly described and the quantity of information justified. If the information/measurements are too sparse or too simple, this achievement level cannot be awarded. If the information/measurements are from a* ***secondary*** *source, then there must be evidence of sampling if appropriate. All sampling processes should be completely described.*

Criterion C: Mathematical processes

When presenting diagrams, students are expected to use rulers where necessary and not merely sketch. A freehand sketch would not be considered a correct mathematical process. When technology is used, the student would be expected to show a clear understanding of the mathematical processes used. All graphs must contain all relevant information. The teacher is responsible for determining the accuracy of the mathematics used and must indicate any errors on the final project. If a project contains no simple mathematical processes, then the first two further processes are assessed as simple.

Achievement level	Descriptor
0	The project does not contain any mathematical processes. *For example, where the processes have been copied from a book, with no attempt being made to use any collected/generated information.* *Projects consisting of only historical accounts will achieve this level.*
1	At least two simple mathematical processes have been carried out. *Simple processes are considered to be those that a mathematical studies SL student could carry out easily, for example, percentages, areas of plane shapes, graphs, trigonometry, bar charts, pie charts, mean and standard deviation, substitution into formulae* ***and any*** *calculations and/or graphs using technology only.*
2	At least two simple mathematical processes have been carried out correctly. *A small number of isolated mistakes should not disqualify a student from achieving this level. If there is incorrect use of formulae, or consistent mistakes in using data, this level cannot be awarded.*
3	At least two simple mathematical processes have been carried out correctly. All processes used are relevant. *The simple mathematical processes must be relevant to the stated aim of the project.*

Achievement level	Descriptor
4	The simple relevant mathematical processes have been carried out correctly. In addition, at least one relevant further process has been carried out. *Examples of further processes are differential calculus, mathematical modelling, optimization, analysis of exponential functions, statistical tests and distributions, compound probability. For this level to be achieved, it is not required that the calculations of the further process be without error. At least one further process must be calculated, showing full working.*
5	The simple relevant mathematical processes have been carried out correctly. In addition, at least one relevant further process has been carried out. All processes, both simple and further, that have been carried out are without error. *If the measurements, information or data are limited in scope, then this achievement level cannot be awarded.*

Criterion D: Interpretation of results

Use of the terms "interpretation" and "conclusion" refer very specifically to statements about what the mathematics used tells us after it has been used to process the original information or data. Discussion of limitations and validity of the processes is assessed elsewhere.

Achievement level	Descriptor
0	The project does not contain any interpretations or conclusions. *For the student to be awarded this level, there must be no evidence of interpretation or conclusions anywhere in the project, or a completely false interpretation is given without reference to any of the results obtained.*
1	The project contains at least one interpretation or conclusion. *Only minimal evidence of interpretations or conclusions is required for this level. This level can be achieved by recognizing the need to interpret the results and attempting to do so, but reaching only false or contradictory conclusions.*
2	The project contains interpretations and/or conclusions that are consistent with the mathematical processes used. *A "follow through" procedure should be used and, consequently, it is irrelevant here whether the processes are either correct or appropriate; the only requirement is consistency.*
3	The project contains a meaningful discussion of interpretations and conclusions that are consistent with the mathematical processes used. *To achieve this level, the student would be expected to produce a discussion of the results obtained and the conclusions drawn based on the level of understanding reasonably to be expected from a student of mathematical studies SL. This may lead to a discussion of underlying reasons for results obtained.* *If the project is a very simple one, with few opportunities for substantial interpretation, this achievement level cannot be awarded.*

Criterion E: Validity

Validity addresses whether appropriate techniques were used to collect information, whether appropriate mathematics was used to deal with this information, and whether the mathematics used has any limitations in its applicability within the project. Any limitations or qualifications of the conclusions and interpretations should also be judged within this criterion. The considerations here are independent of whether the particular interpretations and conclusions reached are correct or adequate.

Achievement level	Descriptor
0	There is no awareness shown that validity plays a part in the project.
1	There is an indication, with reasons, if and where validity plays a part in the project. *There is discussion of the validity of the techniques used* ***or*** *recognition of any limitations that might apply. A simple statement such as "I should have used more information/measurements" is not sufficient to achieve this level. If the student considers that validity is not an issue, this must be fully justified.*

Criterion F: Structure and communication

The term "structure" should be taken primarily as referring to the organization of the information, calculations and interpretations in such a way as to present the project as a logical sequence of thought and activities starting with the task and the plan, and finishing with the conclusions and limitations.

Communication is not enhanced by a large number of repetitive procedures. All graphs must be fully labelled and have an appropriate scale.

It is not expected that spelling, grammar and syntax are perfect, and these features are not judged in assigning a level for this criterion. Nevertheless, teachers are strongly encouraged to correct and assist students with the linguistic aspects of their work. Projects that are very poor linguistically are less likely to excel in the areas that are important in this criterion. Projects that do not reflect the significant time commitment required will not score highly on this assessment criterion.

Achievement level	Descriptor
0	No attempt has been made to structure the project. *It is not expected that many students will be awarded this level.*
1	Some attempt has been made to structure the project. *Partially complete and very simple projects would only achieve this level.*
2	The project has been structured in a logical manner so that it is easily followed. *There must be a logical development to the project. The project must reflect the appropriate commitment for this achievement level to be awarded.*
3	The project has been well structured in accordance with the stated plan **and** is communicated in a coherent manner. *To achieve this level, the project would be expected to read well, and contain footnotes and a bibliography, as appropriate. The project must be focused and contain only relevant discussions.*

Criterion G: Notation and terminology

This criterion refers to the use of correct terminology and mathematical notation. The use of calculator or spreadsheet notation is not acceptable.

Achievement level	Descriptor
0	The project does not contain correct mathematical notation or terminology. *It is not expected that many students will be awarded this level.*
1	The project contains some correct mathematical notation **or** terminology.
2	The project contains correct mathematical notation **and** terminology throughout. *Variables should be explicitly defined. An isolated slip in notation need not preclude a student from achieving this level. If it is a simple project requiring little or no notation and/or terminology, this achievement level cannot be awarded.*

Glossary of command terms

Command terms with definitions

Students should be familiar with the following key terms and phrases used in examination questions, which are to be understood as described below. Although these terms will be used frequently in examination questions, other terms may be used to direct students to present an argument in a specific way.

Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Display information in a diagrammatic or logical form.
Deduce	Reach a conclusion from the information given.
Describe	Give a detailed account.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Find	Obtain an answer showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Plot	Mark the position of points on a diagram.
Show	Give the steps in a calculation or derivation.

Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Notation list

Of the various notations in use, the IB has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IB notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

Students must always use correct mathematical notation, not calculator notation.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	the empty (null) set
U	the universal set
$\cup$	union
$\cap$	intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of
A'	the complement of the set A

$p \wedge q$	conjunction: p and q
$p \vee q$	disjunction: p or q (or both)
$p \veebar q$	exclusive disjunction: p or q (**not** both)
$\neg p$	negation: not p
$p \Rightarrow q$	implication: if p then q
$p \Leftarrow q$	implication: if q then p
$p \Leftrightarrow q$	equivalence: p is equivalent to q
$a^{1/n}$, $\sqrt[n]{a}$	a to the power $\frac{1}{n}$, nth root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$a^{-n} = \frac{1}{a^n}$	a to the power $-n$, reciprocal of a^n
$a^{1/2}$, $\sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geq 0$ then $\sqrt{a} \geq 0$)
$\lvert x \rvert$	the modulus or absolute value of x, that is $\begin{cases} x \text{ for } x \geq 0, x \in \mathbb{R} \\ -x \text{ for } x < 0, x \in \mathbb{R} \end{cases}$
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to
$\ngtr$	is not greater than
$\nless$	is not less than
u_n	the nth term of a sequence
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + \ldots + u_n$

$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + \ldots + u_n$
$f(x)$	the image of x under the function f
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
sin, cos, tan	the circular functions
$\mathrm{A}(x, y)$	the point A in the plane with Cartesian coordinates x and y
$\hat{\mathrm{A}}$	the angle at A
$\mathrm{C\hat{A}B}$	the angle between the lines CA and AB
$\Delta\mathrm{ABC}$	the triangle whose vertices are A, B and C
$\mathrm{P}(A)$	probability of event A
$\mathrm{P}(A')$	probability of the event "not A"
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\bar{x}$	mean of a set of data
μ	population mean
σ	population standard deviation
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
$X \sim \mathrm{N}(\mu, \sigma^2)$	random variable X has a normal distribution with mean μ and variance σ^2
r	Pearson's product–moment correlation coefficient
χ^2	chi-squared